Analysis of a Narrow Capacitive Strip in Waveguide

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***Abstract*—A theoretical determination is made for the susceptance of a narrow conducting strip inserted vertically into a waveguide. The theory is based upon a variational form for the susceptance. A suitable current distribution along the strip is obtained for the variational equation, and is found to be similar to that determined from analysis of backscattering by a cylindrical obstacle irradiated from an incident plane wave. Accurate theoretical results may be obtained using a sinusoidal current distribution having a phase constant of $\pi/2d$, where d is the strip depth. Experimental results agree closely with the theory in the dominant-mode range and also at frequencies below cutoff.**

I. INTRODUCTION

THIS PAPER reports an analysis of the effect of insertion of a narrow infinitesimally thin conducting strip vertically into a rectangular waveguide such that the strip does not extend across the entire height of the guide. The principal feature of the variational method used here is the determination of a suitable form for the current distribution along the strip. The effort to find this distribution was motivated by the need to characterize the obstacle reactance at frequencies below the dominant-mode cutoff frequency as well as in the more usual dominant-mode range.

A variety of capacitive obstacles are in common use in waveguide circuit design, including capacitive windows and circular cylindrical probes or tuning screws inserted vertically into the guide. Use of the narrow strip was considered here, in preference to the more usual obstacles, because it offered a ready means of adjusting the strip insertion, with the likelihood of decreased loss when compared to the capacitive tuning screw.

The variational technique developed by Schwinger [1] has been applied to the narrow inductive strip by Collin [2], and to the circular cylindrical probe in waveguide by Lewin [3], [4] and Collin [2]. The resulting variational expression for the shunt reactance contains a term representing the current distribution along the obstacle; substitution of an approximate expression for this distribution yields a reactance value which provides an upper bound to the exact value. Typically, the current distribution has been assumed constant for the inductive strip; for the probe, the current is taken to have a sinusoidal form, with a free-space phase constant $k_0 = 2\pi/\lambda$, and with a zero at the open end of the probe. Experimental measurements by Al-Hakkak [5] agree with the theoretical results of Collin, provided the probe length does not exceed 0.6 of the guide height. Detinko and Levdikova [6] have considered the case where the current is constant along the probe, in an attempt to reconcile the theory with experimental measurements on long probes.

Characterization in the waveguide cutoff-frequency range was required because the capacitive obstacles were being considered for use in the design of evanescent-mode waveguide filters [7]. Mok [8] has presented expressions for diaphragm reactances in evanescent waveguides, and has reported measurements indicating that a probe obstacle remains capacitive below the waveguide cutoff frequency.

II. THE VARIATIONAL EQUATION

The structure to be analyzed is shown in Fig. 1. The strip is assumed to be infinitesimally thin, and to be sufficiently narrow that the current does not vary appreciably with x, in the range $x_1 \leq x \leq x_2$. Both the strip and the waveguide are assumed to be formed of material having infinite conductivity. The strip is located at $z = 0$.

The derivation of the variational equation follows the procedure set out in Collin [2], with additional complexity introduced by the fact that the current is a function of the coordinate y in the present case.

Considering a dominant-mode incident electric field

$$\mathbf{E}_i = \sin\left(\frac{\pi x}{a}\right) \exp\left(-\Gamma_{10} z\right) \hat{\mathbf{y}}$$

the resulting scattered field may be expressed as

$$\mathbf{E}_s = -j\omega\mu_0 \int_S \bar{\mathbf{G}}(\mathbf{r} \mid \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}')\, d\mathbf{r}' \qquad (1)$$

where the integration is carried out over the strip surface S, and the dyadic Green's function for use here is given by Tai [9] in the form

$$\bar{\mathbf{G}}(\mathbf{r} \mid \mathbf{r}') = G_y(\mathbf{r} \mid \mathbf{r}') \hat{\mathbf{y}}\hat{\mathbf{y}}$$

where

$$G_y(\mathbf{r} \mid \mathbf{r}') = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \frac{(2 - \delta_m)(k_0^2 - m^2\pi^2/b^2) \exp\left(-\Gamma_{nm} \mid z \mid\right)}{abk_0^2\Gamma_{nm}} \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{n\pi x'}{a}\right) \cos\left(\frac{m\pi y}{b}\right) \cos\left(\frac{m\pi y'}{b}\right)$$

with

$$\delta_m = \begin{cases} 1, & \text{when } m = 0 \\ 0, & \text{when } m \neq 0 \end{cases}$$

Manuscript received September 21, 1973; revised November 15, 1973. This work was supported by the National Science Foundation under Grant GK-32370.

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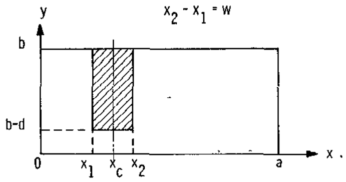

Fig. 1. The thin capacitive strip in rectangular waveguide.

and

$$\Gamma_{nm} = \left(\frac{m^2\pi^2}{b^2} + \frac{n^2\pi^2}{a^2} - k_0^2\right)^{1/2}.$$

This form for the Green's function is based upon the assumption that the scattered field lies in the **y** direction. The total field at any point **r** is given by

$$\mathbf{E}_t(\mathbf{r}) = \mathbf{E}_i(\mathbf{r}) + \mathbf{E}_s(\mathbf{r}) = \sin\left(\frac{\pi x}{a}\right) \exp\left(-\Gamma_{10} z\right)\mathbf{y} + \int_S (-j\omega\mu_0)\,\bar{\mathbf{G}}(\mathbf{r} \mid \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}')\, d\mathbf{r}. \quad (2)$$

The current is assumed to have only a y component, and to be a function only of y on the narrow strip surface S.

Since E_t vanishes on the perfectly conducting surface S,

$$\sin\left(\frac{\pi x}{a}\right) - j\omega\mu_0 \int_S \left[G_y(\mathbf{r} \mid \mathbf{r}')\right]_{z=0} J_y(y')\, dx'dy' = 0, \quad \text{on } S. \quad (3)$$

The reflected dominant mode for $z < 0$ is characterized by a reflection coefficient R. The value of R comes from (1) in the form

$$R = \frac{-j\omega\mu_0}{ab\Gamma_{10}} \int_S J_y(y') \sin\left(\frac{\pi x'}{a}\right) dx'dy'. \quad (4)$$

Substituting for R in (3),

$$(1 + R)\sin\left(\frac{\pi x}{a}\right) = j\omega\mu_0\left(\sum_{n=2}^{\infty}\sum_{m=0}^{\infty} + \sum_{m=1;\ \text{at}\ n=1}^{\infty}\right) \frac{(2-\delta_m)(k_0^2 - m^2\pi^2/b^2)}{abk_0^2\Gamma_{nm}} \cdot \sin\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right) \int_S \sin\left(\frac{n\pi x'}{a}\right)\cos\left(\frac{m\pi y'}{b}\right) \cdot J_y(y')\, dx'dy' \quad (5)$$

which may be put in the form

$$(1 + R)\int_S J_y(y)\sin\left(\frac{\pi x}{a}\right) dxdy = j\omega\mu_0\left[\left(\sum_{n=2}^{\infty}\sum_{m=0}^{\infty} + \sum_{m=1;\ \text{at}\ n=1}^{\infty}\right) \cdot \frac{(2-\delta_m)(k_0^2 - m^2\pi^2/b^2)J_{nm}}{abk_0^2\Gamma_{nm}}\right] \quad (6)$$

where

$$J_{nm} = \int_S\int_S J_y(y)J_y(y')\sin\left(\frac{n\pi x}{a}\right)\sin\left(\frac{n\pi x'}{a}\right)\cos\left(\frac{m\pi y}{b}\right) \cdot \cos\left(\frac{m\pi y'}{b}\right) dxdydx'dy'.$$

The obstacle has a normalized susceptance $j\bar{B}$ given by

$$\bar{Y} = j\bar{B} = \frac{-2R}{1+R}.$$

Using (4) and (6), an expression for the susceptance is found in the form

$$j\bar{B} = 2\left[\int J_y(y)\sin\left(\frac{\pi x}{a}\right)dxdy\right]^2 \Big/ \frac{\Gamma_{10}}{k_0^2}\left[\left(\sum_{n=2}^{\infty}\sum_{m=0}^{\infty} + \sum_{m=1;\ \text{at}\ n=1}^{\infty}\right)\frac{(2-\delta_m)(k_0^2 - m^2\pi^2/b^2)J_{nm}}{\Gamma_{nm}}\right]. \quad (7)$$

Using a method similar to that of Lewin [3, p. 79] this expression may be shown to be stationary for small variations in J_y about its correct value. Use of an approximate form for J_y in (7) yields a lower bound on the true value of the susceptance.

The expression (7) reduces to a form identical to that of Collin [2] if the strip extends across the entire waveguide height and the current is assumed constant along the strip.

III. CURRENT DISTRIBUTION ALONG THE STRIP

Waveguide probe studies [2], [3], [5] have used an approximate current distribution of the form

$$J_y(y) = \sin k_0(y - b + d). \quad (8)$$

Consequently, this distribution was used in (7) for the narrow strip. The resulting values of $\bar{B}$ were found to be in reasonable agreement with the experimental measurements for the dominant-mode propagation frequency range. However, the theoretical values were considerably less than those found by experiment in the dominant-mode cutoff frequency range (shown in Fig. 5).

Use was then made of the modified distribution

$$J_y(y) = \sin k_1(y - b + d) \quad (9)$$

where k_1 is to be determined by substituting in (7) and finding the k_1 value which extremizes the variational expression for $j\bar{B}$. Implicit in this choice is the assumption that the current is zero at the end of the strip where $y = b - d$. This is valid when the end-capacity of the strip can be neglected [3, p. 86]. When d approaches b, the validity of the assumption will be weakened.

Substituting (9) in (7) and evaluating the integrals, we obtain

$$j\bar{B} = C(\cos k_1 d - 1)^2 \Big/ \left\{A(\cos k_1 d - 1)^2 + \sum_{m=1}^{\infty} \left\{\frac{k_1^4 B_m b^4}{(m^2\pi^2 - k_1^2 b^2)^2}\left[(-1)^m \cos k_1 d - \cos\frac{m\pi(b-d)}{b}\right]^2\right\}\right\} \qquad (10)$$

where

$$A = \sum_{n=2}^{\infty} \left\{\frac{1}{n^2\Gamma_{n0}}\left[\cos\left(\frac{n\pi x_1}{a}\right) - \cos\left(\frac{n\pi x_2}{a}\right)\right]^2\right\}$$

$$B_m = \sum_{n=1}^{\infty} \left\{\frac{2(k_0^2 - m^2\pi^2/b^2)}{k_0^2\Gamma_{nm}n^2}\left[\cos\left(\frac{n\pi x_1}{a}\right) - \cos\left(\frac{n\pi x_2}{a}\right)\right]^2\right\}$$

$$C = \frac{2}{\Gamma_{10}}\left[\cos\left(\frac{\pi x_1}{a}\right) - \cos\left(\frac{\pi x_2}{a}\right)\right]^2.$$

Putting $(\partial/\partial k_1)(j\bar{B}) = 0$ yields the equation

$$b^4 k_1^3(\cos k_1 d - 1)\, f(k_1) = 0 \qquad (11)$$

where

$$\begin{aligned} f(k_1) = \sum_{m=1}^{\infty} \Bigg\{&\left[\frac{B_m}{(m^2\pi^2 - k_1^2 b^2)^2}\right] \\ &\cdot\left[(-1)^m \cos(k_1 d) - \cos\left(\frac{m\pi(b-d)}{b}\right)\right] \\ &\cdot\Bigg\{2k_1 d \sin(k_1 d)\cos\left(\frac{m\pi(b-d)}{b}\right) - 4(-1)^m \\ &\cdot\cos^2(k_1 d) + 4(-1)^m\cos(k_1 d) + 4\cos(k_1 d) \\ &\cdot\cos\left(\frac{m\pi(b-d)}{b}\right) - 4\cos\left(\frac{m\pi(b-d)}{b}\right) \\ &- 2(-1)^m k_1 d \sin(k_1 d) - \frac{4k_1^2 b^2}{m^2\pi^2 - k_1^2 b^2} \\ &\cdot[\cos(k_1 d) - 1]\Bigg[(-1)^m\cos(k_1 d) \\ &- \cos\left(\frac{m\pi(b-d)}{b}\right)\Bigg]\Bigg\}\Bigg\}. \end{aligned} \qquad (12)$$

The solution $\cos k_1 d = 1$ to (11) is inappropriate to our purpose since it gives $j\bar{B} = 0$. We are therefore led to seek solutions to the equation $f(k_1) = 0$. Despite its apparent complexity, this equation is readily solved numerically, taking advantage of the fact that the infinite series are all rapidly convergent and can be approximated by a small number of terms. Using a half-section method, with an error interval of 0.03, the infinite series could be truncated with $N \leq 20$, $M \leq 20$.

The resulting k_1 values are shown for a typical set of electrical and geometric parameters in Fig. 2. It is evident that k_1 is only slightly dependent on frequency, position, and width, and that its value is quite different from k_0.

The variation of k_1 with d can be approximated by the equation $k_1 = \pi/2d$ with good accuracy. Thus the variational solution for k_1 yields a sinusoidal current distribution which proceeds from zero at the open end of the strip to a maximum at the point close to $y = b$, where the strip meets the waveguide wall.

Use of (9) with $k_1 = \pi/2d$ reduces (10) to the form

$$j\bar{B} = C \Big/ \left\{A + \sum_{m=1}^{\infty} \left\{\left[\frac{B_m b^4}{(4m^2d^2 - b^2)^2}\right]\cos^2\left(\frac{m\pi(b-d)}{b}\right)\right\}\right\}. \qquad (13)$$

This distribution is reminiscent of that obtained from analysis of backscattering by a cylindrical obstacle irradiated from a broad-side incident plane wave. Tai [10] has shown that accurate results may be obtained for a long thin wire of length $2l$ centered at the origin and coinciding with the z axis by use of the current distribution

$$I(z) = I_0(\cos k_0 z - \cos k_0 l)$$

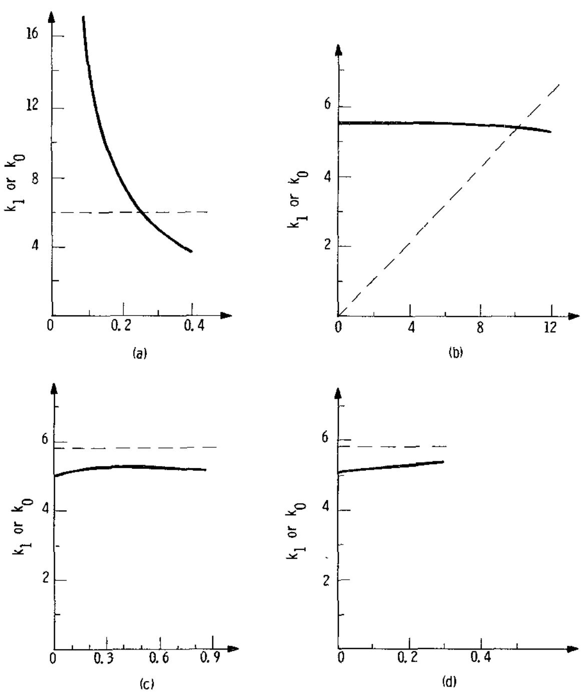

Fig. 2. The free-space parameter k_0 and the variational parameter k_1 shown as a function of (a) strip depth d (in); (b) frequency f (GHz); (c) strip-center location x_c (in); (d) strip width w (in). The following values are used, except for the quantity being varied: $w = 0.15$ in; $d = 0.3$ in; $f = 11.0$ GHz; $x_c = a/2$. The solid line indicates k_1 and the broken line k_0; the dimensions of k_0 and k_1 are reciprocal inches. For these calculations, $a = 0.90$ in; $b = 0.40$ in.

which proceeds uniformly from zero at the ends to a maximum at the center, for $k_0 l < \pi$. He also shows that the accuracy obtained with this distribution is reduced when l takes values satisfying $\tan k_0 l = k_0 l$.

Arguing by analogy, it might thus be anticipated that a suitable alternative to use of (9) for the narrow strip, with $k_1 = \pi/2d$, would be use of

$$J_y(y) = \cos k_0(y - b) - \cos k_0 d. \qquad (14)$$

For conventional waveguide, $k_0 b < \pi$ for frequencies extending to the upper end of the dominant-mode region. Substitution of (14) in (10) yields an expression for the normalized susceptance in the form

$$j\bar{B} = C\left[\frac{1}{k_0}\sin(k_0 d) - d\cos(k_0 d)\right]^2 \Bigg/ \left\{A\left[\frac{1}{k_0}\sin(k_0 d) - d\cos(k_0 d)\right]^2 + \sum_{m=1}^{\infty} \frac{B_m D_m k_0^2 b^4}{(m^2\pi^2 - k_0^2 b^2)^2}\right\} \qquad (15)$$

where

$$D_m = \left(\frac{k_0^2 b^2}{m^2\pi^2}\right)\cos^2(k_0 d)\sin^2\left(\frac{m\pi(b-d)}{b}\right) + \left(\frac{k_0 b}{2m\pi}\right)\sin(2k_0 d)\sin\left(\frac{2m\pi(b-d)}{b}\right) + \sin^2(k_0 d)\cos^2\left(\frac{m\pi(b-d)}{b}\right).$$

IV. EXPERIMENTAL MEASUREMENT PROCEDURE

The susceptance is readily measured in the dominant-mode range using conventional slotted-line techniques. However, this method is inapplicable to measurements in the frequency range below cutoff. Mok [11] has described a procedure using a slotted line together with a waveguide filled with dielectric material; the range of frequencies below cutoff over which measurements can be made depends on the value of the dielectric constant, and the accuracy decreases as the frequency decreases.

The technique used here depends upon resonance of the obstacle with a section of waveguide below cutoff, and is based on the filter design theory of Craven and Mok [7]. A waveguide section of length $2l$, with a shunt strip obstacle at the center of that length, is shown in Fig. 3(a), and its equivalent circuit illustrated in Fig. 3(b). The characteristic impedance of the line is given by jX_0, where

$$X_0 = 120\pi\left(\frac{b}{a}\right)\left[\left(\frac{\lambda}{\lambda_c}\right)^2 - 1\right]^{-1/2}$$

for evanescent TE_{10}-mode excitation.

The propagation constant Γ is now real, given by

$$\Gamma = \frac{2\pi}{\lambda}\left[\left(\frac{\lambda}{\lambda_c}\right)^2 - 1\right]^{1/2}.$$

The susceptance to be measured is expressed by the unnormalized value jB. The circuit of Fig. 3(b) may be rearranged into that shown in Fig. 3(c), where J-inverter elements have been extracted from each waveguide section. Comparing Fig. 3(b) and (c),

$$B_L = -\frac{\coth \Gamma l}{X_0}.$$

The circuit is resonant when $B + 2B_L = 0$, i.e.,

$$BX_0 = 2\coth \Gamma l.$$

Hence use of a known length of waveguide permits determination of the normalized value of B through measurement of the resonant frequency resulting from a specific strip insertion. Using this method, the B value is determined only at the resonant frequency, which varies with change in strip depth because l is being held constant.

V. EXPERIMENTAL AND THEORETICAL RESULTS

Measurements carried out with conventional X-band waveguide gave the results shown in Fig. 4 for most of the dominant-mode range. Shown also are the theoretical results obtained for the two strips using the current distribution of (9) with k_1 determined from (12). The theoretical results obtained using (8) are not shown here, since the values were only slightly less than those obtained using (9); similar remarks apply to B values obtained using (14).

Results obtained at frequencies below cutoff are presented in Fig. 5. In this case, the results show jB as a function of depth for a variety of frequencies, each of which is defined by resonance of the obstacle and the waveguide of length $l = 1.445$ in. It is evident that use of the current distributions specified by (9) or (14) yields values much closer to the measured values that are obtained by use of (8).

These results, and those of Al-Hakkak [5], may be explained by noting that above cutoff the current distribu-

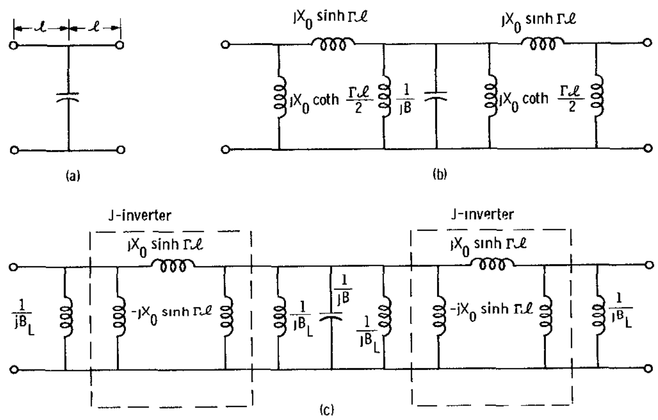

Fig. 3. A waveguide section below cutoff with a capacitive obstacle at the center of the guide. (a) Basic geometry. (b) Equivalent circuit. (c) Equivalent circuit modified to show J inverters.

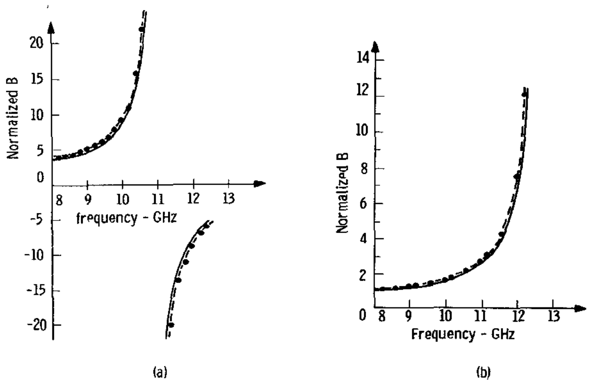

Fig. 4. Normalized values of strip susceptance as a function of frequency for a centered strip, excited at frequencies in the dominant-mode region. The solid line shows theoretical results found using (9) and (12), and the broken line shows experimental measurements. (a) For $w = 0.156$ in, $d = 0.307$ in. (b) For $w = 0.081$ in, $d = 0.238$ in.

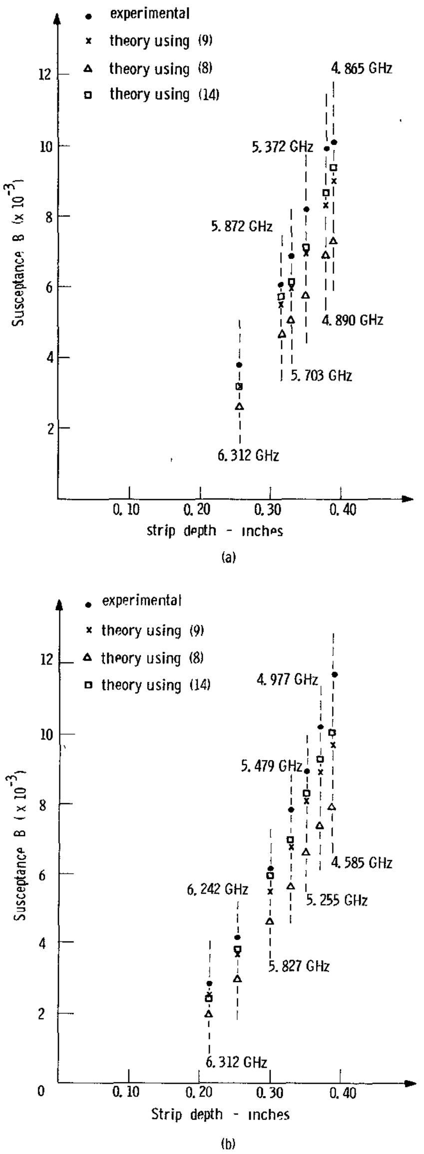

Fig. 5. The strip susceptance as a function of strip depth for a centered strip, excited at frequencies in the waveguide cutoff region. The resonant frequency of the waveguide–obstacle combination is shown for each depth at which measurement occurred. (a) For $w = 0.097$ in. (b) For $w = 0.16$ in.

tions given by (8), (9), and (14) will be very similar until the probe depth $d > \lambda/4$, for which depth (8) gives a current maximum at $y < b$ while (9) and (14) give the maximum at $y = b$. Consequently, as Al-Hakkak points out, (8) gives inaccurate values for $d > \lambda/4$. Below cutoff frequency, $k_0 d \ll \pi/2$, and (8) gives a current distribution which is almost of constant slope, quite different from the sinusoidal distribution of (9) and the almost-sinusoidal form of (14). Hence the results obtained below cutoff using (9) or (14) differ appreciably from those obtained using (8).

The principal source of error in the measurements occurs in determination of the insertion depth of the narrow thin strip into the waveguide. The below-cutoff measurements were susceptible to this error since the depth was changed for each experimental point. The resonant circuit formed by the strip and the below-cutoff guide was fed by a coaxial line; consequently, a further source of error arises from neglect of the discontinuity reactances associated with the adapter from coaxial line to waveguide.

The resonance effect obtained with the strip used for Fig. 4(a) may be explained by reference to (10). Above the cutoff frequency, C is imaginary and negative. At the low end of the frequency range, the second term in the denominator is negative and greater in magnitude than the first term; as a result, the susceptance is capacitive. With increasing frequency or insertion depth, a frequency can be found, such that the two denominator terms are equal and of opposite sign so that resonance occurs, beyond which the susceptance is inductive.

At frequencies below cutoff, C is real, with the result that the normalized value of $\bar{B}$ given by (10) is imaginary (i.e., $j\bar{B}$ is real and negative). However, in this frequency range the waveguide characteristic impedance is inductive.

Using (10) in conjunction with (9), the effect of variation in strip dimensions and location can be studied. Fig. 6 shows the variation in $\bar{B}$ with increase in insertion depth for representative frequencies below and above waveguide cutoff. The effect of increase in strip width for a centered strip is shown in Fig. 7; as the width increases,

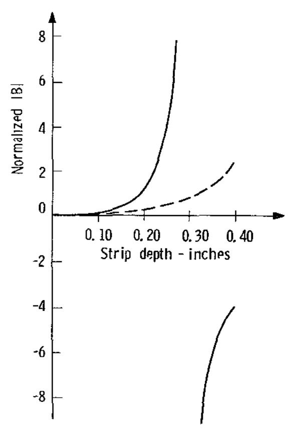

Fig. 6. Normalized values of strip susceptance as a function of depth, calculated for a centered strip with $w = 0.12$ in. The solid line is for $f = 11.0$ GHz, and the broken line for $f = 5.0$ GHz.

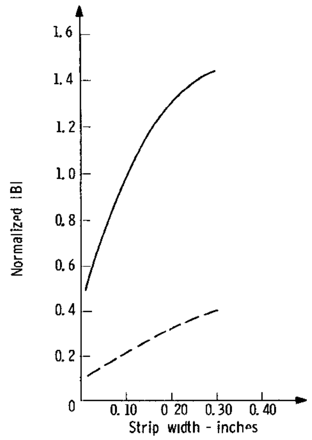

Fig. 7. Normalized values of strip susceptance as a function of strip width, calculated for a centered strip with $d = 0.20$ in. The solid line is for $f = 11.0$ GHz and the broken line for $f = 5.0$ GHz.

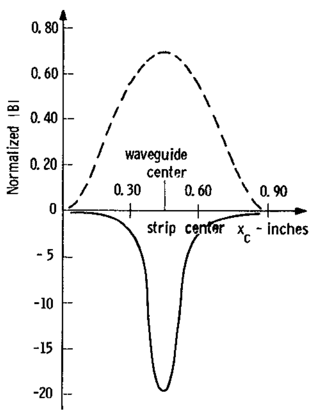

Fig. 8. Normalized values of strip susceptance as a function of strip location, calculated for a strip with $w = 0.10$ in and $d = 0.30$ in. The solid line is for $f = 11.0$ GHz and the broken line for $f = 5.0$ GHz.

the assumption that the current distribution is independent of x will require closer examination. Movement of a narrow strip transversely in the x direction across the waveguide yields the curve illustrated in Fig. 8; when the strip is close to the waveguide sidewalls, the current distribution will require modification because of the capacitance between the strip and the proximate wall.

VI. CONCLUSIONS

The variational expression developed here, together with a current distribution based either upon extremization of the variational expression for current or upon use of the analogy to backscattering of a cylindrical obstacle by an incident plane wave, provides an accurate characterization for the thin narrow strip inserted partially into waveguide. A feature of the analysis is its applicability at frequencies below that at which the dominant mode propagates without attenuation.

The simplest computation yielding accurate results is given by (13). Results obtained using (8) are inaccurate below the cutoff frequency, or at high frequencies or large probe depths such that $d > \lambda/4$.

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